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INTERACTION OF RADIATION AND MATTER IN A PLASMA

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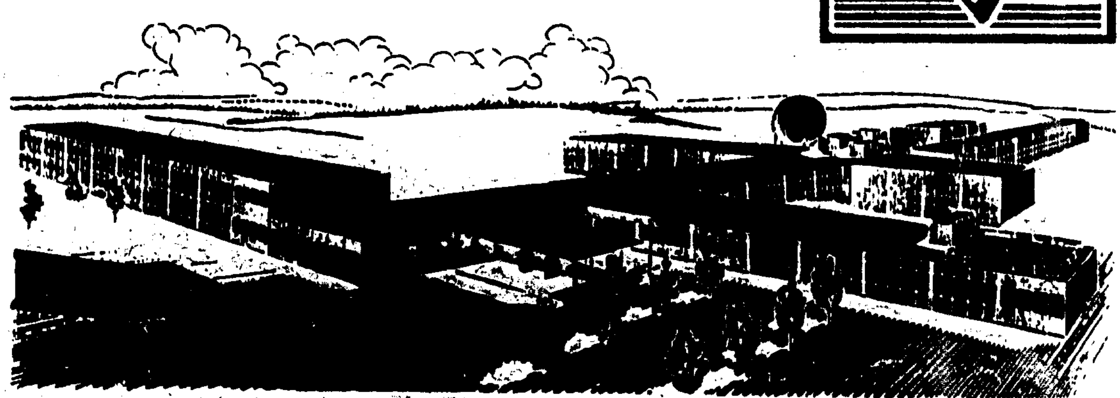
YALE UNIVERSITY
NEW HAVEN, CONNECTICUT

FEBRUARY 1963

AERONAUTICAL RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE



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AERONAUTICAL RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

FOREWORD

This interim technical report was prepared by Yale University Observatory, New Haven, Connecticut, on Contract AF 33(657)-7271 for the Aeronautical Research Laboratories, Office of Aerospace Research, United States Air Force. The research reported herein was accomplished on Task 7073-01, "High Energy Plasma Generation and Control" of Project 7073, "Research on Plasma Dynamics" under the technical cognizance of Dr. Wolfgang Braun of the Plasma Physics Research Laboratory of ARL.

ABSTRACT

This report describes a part of a theoretical investigation of the theory of the emission and absorption of radiation from a fully ionized plasma in the presence of a magnetic field. Specifically, the bremsstrahlung radiation and departures from L. T. E. are discussed.

INTRODUCTION

The work carried out under this contract aims at developing a theory of emission and absorption of radiation in fully ionized plasmas the presence of a magnetic field. The pertinent radiation mechanisms are bremsstrahlung and cyclotron radiation. A general theory of these combined mechanisms must take into account several different problems which determine in each case the line of attack.

At present, more or less explicit theories are available for special cases which are to be combined and expanded to cover the full range of physical possibilities. It was decided to attack the following problems in turn:

1. Emission and absorption of bremsstrahlung following Coulomb interactions is influenced by the presence of a magnetic field, whereas at the same time the cyclotron line is broadened by the Coulomb interactions. A theory of the combined effects was to be developed, neglecting for the time being refractive indices differing from unity, departures from thermodynamic equilibrium and relativistic corrections. 2. The influence of the refractive index was to be considered. This effect is a manifestation of the collective behavior of the plasma. Since a systematic theory is still missing, it was judged most advisable to begin work with simplest case, namely, the influence of shielding corrections on the bremsstrahlung cross section. 3. Departures from thermodynamic equilibrium do not affect radiative interactions of single particles, but only emission and absorption of the plasma as a whole. Since this is essentially a transfer problem, it could be separated from the remainder of the analysis. 4. Relativistic corrections in a general manner are very cumbersome to treat. It was therefore, decided to treat relativistic corrections in each of the general cases under study separately, and at a later stage of the investigation.

In the period covered by this report, a general theory of bremsstrahlung and cyclotron radiation for unity refractive indices was developed and is briefly described in Section 1. The theory of bremsstrahlung emission was expanded to include refractive indices differing from unity, leading to extensive numerical work which is now under way; c.f. Section 2. Departures from thermodynamic equilibrium were treated in a very general manner, and an exhaustive formulation for continuous energy developed. The results are summarized in Section 3.

1. RADIATIVE COULOMB INTERACTION IN THE PRESENCE OF A MAGNETIC FIELD.

The purpose of this study is to obtain a theory of Coulomb interactions in the presence of a magnetic field. The essential parameter is the collision cross section of the free electrons with ions in the magnetic field. The method to be followed is to compute from the wave functions of free electrons in a magnetic field the matrix elements for dipole radiation, where the Coulomb interactions are treated as perturbations of the time varying part and of the space varying part of the wave functions separately.

The statistical superposition of these perturbations is carried out in the manner given by Lindholm[1] and leads directly to the expressions for line shift and half width. The two effects are separable. If one neglects the effect of the Coulomb interactions on the space depending part of the wave functions (adiabatic case in the impact theory of line broadening), only a certain fraction of the cyclotron line's half width and, for that matter, of the scattering cross section of free electrons in a magnetic field, is obtained. However, a line shift is predicted which does not follow from the non-adiabatic case, i.e., the effect of the

Coulomb interactions on the space depending part of the wave functions.

In the following, the adiabatic treatment is reviewed briefly for which definitive results are now available. We mention, however, that preliminary results indicate that the total cross section following from the non-adiabatic treatment does not depend critically on the magnetic field, since in most cases the dependence on the magnetic field strength is logarithmic. Detailed computations covering the general case are scheduled for the following research period.

The spatial part of the wave functions reads[2]

$$\psi_{nks} = (2\pi L)^{-1/2}\, e^{i(kz+(n-s)\phi)} \sqrt{2\gamma} \times (n!\,s!)^{-1/2}\, e^{-\rho/2} \rho^{(n-s)/2}\, Q_s^{n-s}(\rho)\,. \tag{1}$$

The normalization length L is arbitrary and does not appear in the final result. The parameter

$$\gamma = eH/2c\hbar \tag{2}$$

contains the magnetic field. ρ is defined by

$$\rho = \gamma r^2 \tag{3}$$

$\vec{r}$ being the radius vector. k is a continuous variable. s is integer and varies from 0 to ∞, n-s varies from $-\infty$ to n.

$$Q_s^{n-s}(\rho) = (-1)^s \sum_{\mu=0}^{s} (-1)^{\mu} \frac{s!\,n!\,\rho^{s-\mu}}{\mu!\,(s-\mu)!\,(n-\mu)!} \tag{4}$$

is the generalized Laguerre polynomial.

So far, we have considered the wave function of electrons subject only to a constant magnetic field. We now proceed to include the effects of interactions between the electron and positive ions in the two particles approximation. The interaction adds a time and space varying part $H(r, t)$ to the Hamiltonian in the Schrödinger equation which now reads

$$\left[-\frac{\hbar^2}{2m}\left(\vec{p}-\frac{e}{c}\vec{A}\right)^2+H(\vec{r}_1 t)\right]\psi=-i\hbar\,\frac{\partial\psi}{\partial t}\quad. \tag{5}$$

In Eq (5), ψ is the wave function of the electron, $\vec{p}$ the momentum operator. The vector potential $\vec{A}$ and the magnetic field $\vec{H}$ are related by

$$\Delta\times\vec{A}=\vec{H} \tag{6}$$

Assuming that at the initial time there are no perturbations present, we have as solution the value of ψ from Eq (1). The time varying part can be written in the form exp [iEt/h].

The problem of solving Eq (5) is exceedingly complex. The following simplifications were, therefore, introduced[3]. Firstly, we represent the entire set of possible electron-ion interactions by the same electron wave function, but by varying ion positions. The wave function of the electron can then be chosen as a function which is sharply peaked at the Larmor radius. Mathematically, this implies that

$$S = 0 \tag{7}$$

or, in physical terms, that the origin is chosen coincident with the center of the orbit.

Secondly, as already mentioned above, we assume that the effect of the electron ion interaction is to change the time varying part of the wave functions only (adiabatic case). The classical counterpart is to neglect a spatial change in the orbit of the particle due to the interaction. If there were no magnetic fields present, this orbit would be a straight line.

Lindholm[1] has developed a theory for the broadening of spectral lines in the case of interactions producing a "random phase change"[4]. This type of interaction produces a certain change in the eigenfrequency of the oscillator, whereby in a large number of interactions the distribution of frequency modulations is random. In essence, the postulated randomness amounts to saying that the interactions are uncorrelated, and that their effects, therefore, are additive. It is not the scope of this report to review the several steps leading to Lindholm's final result which reads for the spectrum of the cyclotron line

$$I_\omega d_\omega \propto u_1 \left[u_1^2 + (\omega - \omega_e - u_2)^2\right]^{-1} d\omega . \tag{8}$$

The line contour represented by Eq (8) is very close to a Lorentzian form, except that in addition to the classical broadening represented by the parameter u_1 there occurs a shift of the resonance frequency

$$\omega_c = eH/mc \tag{9}$$

which is given by u_2.

Half-width u_1 and shift u_2 were calculated with the aid of the wave functions of Eq (1) and the Coulomb perturbations in Schrödinger's Eq (5), subject to the simplifications outlined above. The results are given in Table 1 (u_1) and Figure 1 (u_2). Whereas, the total cross section will differ from u_1 by the non-adiabatic contribution, the line shift is the definitive result. For a combination of ion number density $n_i = 10^{10}$ cm^{-3} and $H = 10^3$ Gauss, the line shift is about 3 percent of the gyrofrequency. Hence, it should be observable under suitable experimental conditions.

2. BREMSSTRAHLUNG CROSS SECTIONS FOR FREQUENCIES CLOSE TO THE PLASMA FREQUENCY

The conventional theory of bremsstrahlung interactions assumes that the refractive index (ri) is effectively unity[5]. Since (in the absence of a magnetic field), the r. i. (I, Eq 77)

$$n = \left[1 - \omega_p^2/\omega^2\right]^{1/2} \tag{10}$$

where ω_p is the plasma frequency, the condition of r. i. unity requires

$$\omega \gg \omega_p \tag{11}$$

On the other hand, the inequality (11) is required in order to justify the use of a pure Coulomb potential for the electron ion interaction without shielding corrections. In fact, the departures of the r. i. from unity are due to the shielding effects of the plasma electrons on the ion's potential[6]. Hence, accounting for values of $n \neq 1$ means to include shielding effects in the calculation of bremsstrahlung cross sections.

In first approximation, the shielding effects can be included by cutting off the Coulomb potential at the Debye distance[6], and by using this cut-off potential instead of the pure Coulomb potential in the conventional classical calculation of the bremsstrahlung cross sections.

This calculation must then be altered in two ways. Firstly, the time integration in the evaluation of the Fourier components I, Eqs (8) and (9) must be terminated at the finite times when the electron enters and leaves the Debye sphere, respectively. Secondly, only interactions of electrons passing the ion at distances (collision parameters) less than the Debye distance are counted for the average cross section.

Making use of the formulation developed in I, we have in the straight line approximation[I, Section 4] which is valid throughout the r.f. region and, thus, in the neighborhood of the plasma frequency,

$$\ddot{x}(\omega) = \frac{2e^2}{\pi m v_0 b} \int_{-u_0}^{+u_0} \cos(\Omega \epsilon u) \left[1 + u^2\right]^{-3/2} du \tag{12}$$

and

$$\ddot{y}(\omega) = -\frac{2e^2}{\pi m v_0 b} \int_{-u_0}^{+u_0} \sin(\Omega \epsilon u) \left[1+u^2\right]^{-3/2} u\,du, \tag{13}$$

where

$$u_0 = \left[\epsilon_m^2/\epsilon^2 - 1\right]^{1/2} \tag{14}$$

ϵ, the eccentricity of the hyperbolic orbit, and the collision parameter b are related for a given electron velocity v_0 by

$$\epsilon^2 - 1 = b^2 \left[\frac{m v_0^2}{Z e^2}\right]^2 = \left[\frac{b\omega}{v_0 \Omega_0}\right]^2 \tag{15}$$

ϵ_m follows from ϵ by inserting the Debye distance

$$b_m = (kT)^{1/2}/2\pi^{1/2} e N_e \tag{16}$$

[c.f. I, Eq 70] for b. Since $\epsilon_m^2 >> 1$,

$$\epsilon_m \approx b_m \left(\frac{m v_0^2}{Z e^2}\right). \tag{17}$$

The integrals (12 and 13) were computed numerically. The spectrum coefficient

$$Q_\omega = \frac{2e^2}{3c^3} \left\{ \left[\ddot{x}(\omega)\right]^2 + \left[\ddot{y}(\omega)\right]^2 \right\} \tag{18}$$

[I, Eq 71] must then be averaged with respect to all collision parameters or, what amounts to the same, with respect to the eccentricities ϵ. The pertinent average reads [I, Eq 59]

$$\pi \langle Q_\omega \rangle_b \, d\omega = 2\pi N_i v_0 \pi \int_0^{b_m} Q_\omega \, b\, db \cdot d\omega \tag{19}$$

Here, N_i is the number of ions per cm^3. Note the finite upper limit due to the shielding of the ion potential.

Preliminary results for a few test cases show that the average spectrum coefficient is a smooth curve decreasing rapidly for frequencies above the plasma frequency. This behavior is expected on general grounds and agrees, at least qualitatively, with the results of Dawson and Oberman[6].

3. DEPARTURES FROM LTE

The importance of departures from local thermodynamic equilibrium (LTE) was first recognized in connection with certain astrophysical problems, in particular, in connection with the interpretation of the spectrum of the outer solar atmosphere[7]. Stellar plasmas are, in the observable layers, rather weakly ionized, so that the major part of the radiation spectrum consists of spectral lines, i.e., emissions and absorptions involving only bound states. Departures from LTE thenaffect primarily these bound states, with the free energy states in LTE. In physical terms, this statement implies that the free energy states are distributed according to a Maxwell distribution, whereas, emission and absorption are connected by Kirchhoff's law

$$\epsilon_\omega = \kappa_\omega B_\omega . \tag{20}$$

ϵ_ω is the emission per cm^3, sec and steradian, κ_ω is the absorption coefficient per cm, B_ω is Planck's function.

In many domaines of laboratory physics, however, plasmas are completely ionized, and the population of bound states can be altogether neglected. Then, only the free electrons are involved in radiative interactions, and departures from LTE in the radiation field have their counterpart in departures from a Maxwellian distribution of the free energy states.

The purpose of this outline is to give a short description of a general formulation of departures from LTE in free-free emission and absorption. We restrict our considerations to steady states, where the change in the population of states is (during observational times) small compared with the populations themselves. The effects of non-radiative interactions are of minor importance for the problems treated presently, since the conclusions of this Section do not depend on the presence or absence of non-radiative interactions. A steady state, on the other hand, is the case of greatest interest under laboratory conditions.

Radiative interactions affect the particle distribution and the spectrum of the radiation field. There are, therefore, always two sets of conditions to be considered, derived from the behavior of the radiation field, and from the behavior of the particles. Neglecting one set of conditions does mathematically not lead to any contradiction, but leads to physically invalid solutions. This fact has been pointed out previously[8].

The steady state condition for the particle states implies that the number of transitions leading into a given state of energy E_o from all states above (E_1) and below (E_2) must equal the number of transitions out of E_o. Using the analysis published previously[9], we find the steady state condition

$$\int_{E_0}^{\infty} dE \cdot Q(E_1,\Omega)\left\{E_1,\Omega\right\}_{E_0=\text{const}} + \int_{0}^{E_0} dE_2 \cdot Q(E_0,\Omega)\left\{E_0,\Omega\right\}_{E_0=\text{const}} = 0 \tag{21}$$

where

$$\Omega = E_1 - E_0 = E_0 - E_2 = \hbar\omega \tag{22}$$

The bracket symbol is defined by

$$\left\{E,\Omega\right\} \equiv N(E) + I_\omega \cdot \frac{4\pi^3\hbar c^2}{\Omega^3}\left[N(E) - \frac{E}{E-\Omega}\,N(E-\Omega)\right] \tag{23}$$

which contains spontaneous and induced emissions, and absorptions, already reduced[9] to the probability of spontaneous emission

$$Q(E,\Omega)\,dE\,d\Omega\,\left[\text{sec}^{-1}\ \text{sterad}^{-1}\right]. \tag{24}$$

$I_\omega\,d\Omega$ is the specific intensity of the impinging radiation field (erg/cm^2 sec steradian), N(E) is the (arbitrary) number distribution of electrons per energy interval and cm^3.

If no additional conditions are imposed on the system (see below), the consideration of the radiation fields adds the conservation law of energy as a kind of boundary condition. In the above formalism, this condition reads

$$0 = \int_0^\infty dE_0 \int_{E_0}^\infty dE_1 \cdot Q(E_1,\Omega)\{E_1,\Omega\} = \int_0^\infty dE_0 \int_0^{E_0} dE_2 \cdot Q(E_0,\Omega)\{E_0,\Omega\} \tag{25}$$

The set of Eqs (21) and (25) has the trivial solution

$$\{E,\Omega\} \equiv 0 \quad \text{for all} \quad E,\ \Omega \tag{26}$$

It can be easily shown that Eq (26) is equivalent to LTE, i.e., to a system where $N(E)$ is Maxwellian, I_ω is Planckian.

It can also be shown with a little algebra that in a system which is optically thick in the whole spectrum, i.e., in a system where[9]

$$I_\omega = S_\omega = \epsilon_\omega / \kappa_\omega \tag{27}$$

(S_ω is the source function), everywhere, the conservation law of total radiative energy Eq (25) is replaced by the much stronger condition that

$$\int_0^{E_0} dE_2 \cdot Q(E_0,\Omega)\{E_0,\Omega\} = \int_{E_0}^\infty dE_1 \cdot Q(E_1,\Omega)\{E_1,\Omega\} = 0 \tag{28}$$

The only general solution of Eq (28) is the trivial solution (26) for LTE.

Hence, we conclude that a plasma which is optically thick in the whole continuum cannot depart from LTE under stationary conditions. Since Eq (28) does not depend on the presence of non-radiative transitions, the above stated conclusion is valid generally.

At present, systems are investigated in which Eq (27) holds only in certain regions of the spectrum.

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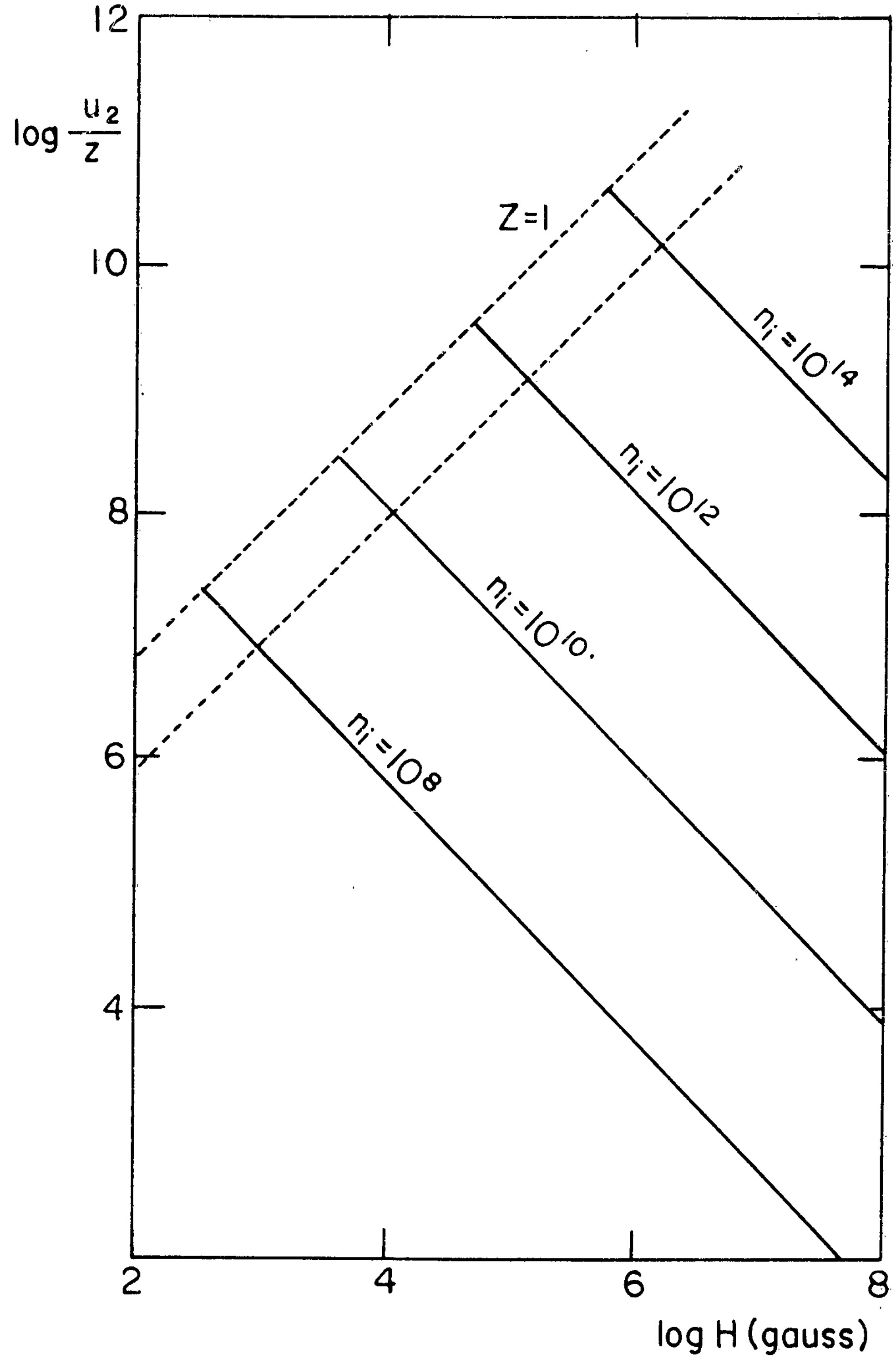
12
10
8
6
4
2
4
6
8
log u2/z
log H (gauss)
Z=1
ni=10^14
ni=10^12
ni=10^10.
ni=10^8

FIG. 1

TABLE 1

$N_i = 10^{10}$,	$H = 10^3$	u_1/ω_c [10^{-6}]
	$T = 10^4$	1.4
	$T = 10^5$	.044
	$T = 10^6$	.0014
$N_i = 10^{10}$,	$H = 10^6$	u_1/ω_c [10^{-9}]
	$T = 10^4$	1.4
	$T = 10^5$	.044
	$T = 10^6$	.0014
$N_i = 10^{14}$,	$H = 10^6$	u_1/ω_c [10^{-5}]
	$T = 10^4$	1.4
	$T = 10^5$	.044
	$T = 10^6$	.0014

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